# Formalizing Knowing Why (1)

Yanjing Wang, Chao Xu

Chao Xu

Department of Philosophy, Peking University

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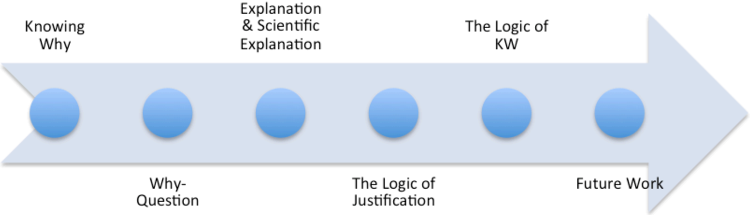
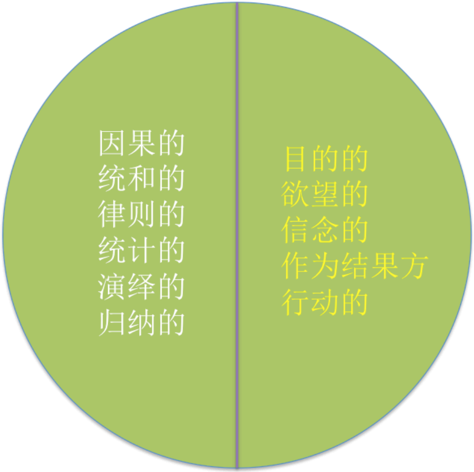

Knowing Why
Why-Question
Explanation & Scientific Explanation
The Logic of Justification
The Logic of KW
Future Work

# §1 Why-Question

将 Why-Question 按 Answer 类型分类:



### Example (因果)

Q: 为什么地上湿了？

A: 因为下雨了。

### Example (律则 (Nomological))

Q: 为什么天空是蓝色的？

A:

1. 大气主要由氮和氧的分子构成（初始或边界条件）
2. 瑞利定律：任意波长的光被一种气体分子散射的量取决于其“散射系数” $1/\lambda^4$
3. 大气中的分子会按瑞利定律散射照射到它上面的光。
4. 蓝光的波长 400nm, 红光 640nm.

### Example (演绎 -律则 (Inductive-Nomological))

Q: 为什么苏格拉底会死？

A:(1) 所有人都会死

(2) 苏格拉底是人

### Example (统计的)

Q: 为什么 R 女士在最近的选举中投了左翼候选人的票？

A:(1)80% 的选民依照其双亲（性别对应，女儿按照母亲、儿子按照父亲）的选择进行投票。（已得到很好地统计概括）

(2)R 女士的母亲投票投给了左翼候选人。

### Example (归纳的)

Q: 为什么 R 女士在最近的选举中投了左翼候选人的票？

A: 因为 R 女士所在的社区的其他所有选举者都将票投给了左翼.

### Example (目的、欲望)

Q: 为什么他如此努力学习？

A: 因为他想取得好成绩。

### Example (信念)

Q: 为什么他如此努力学习？

A: 因为他觉得努力学习是一个学生的本分。

### Example (目的、作为结果方的行动)

Q: 为什么央行要提高利率？

A: 因为央行为了抑制通货膨胀。

# What are answers to Why-Questions?

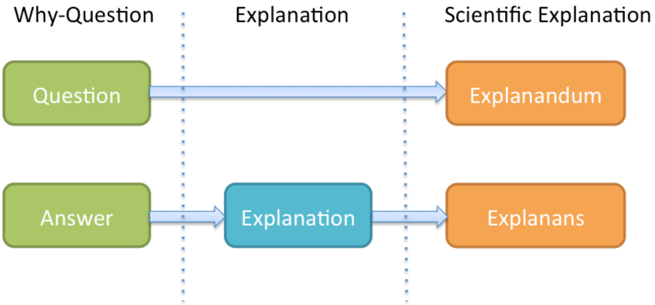
- Why: used to ask or talk about the reason for something
- Explanation: the reasons you give for why something happened or why you did something.
- Cause: a person, event, or thing that makes something happen.
- Reason: why someone decides to do something, or the cause or explanation for something that happen.

*LONGMAN Dictionary of Contemporary English(the Forth Edition)*

# §2 Explanation & Scientific Explanation

*Since explanations can often be thought of as answers to why-questions, we also discuss some topics in the theory of explanation.*

*An Approach To Why-Questions(Antti Koura,1988)*



# Scientific Explanation

科学说明：传统科学哲学寻找任何科学说明都应该满足的一份条件清单。当所有条件都被满足时，此清单保证了一种说明的科学适当性。这个清单上的条件，单个来看都是必要条件，合起来看就是充分条件。

科学说明概念的一种显示定义或者“精释（*explication*，详细解释）”，在增加科学适当性的方向上，可以为评定说明或者改进说明提供一种类似石蕊指示剂的检验或者标尺，从而担负开处方的任务。

*Scientific Explanation (Alex, 刘华杰译,2002)*

# Hempel Deductive-Nomological(DN) model

R1: The explanandum must be a logical consequence of the explanans.

R2: The explanans must contain general laws, and these must actually be required for the derivation of the explanandum.

R3: The explanans must have empirical content; i.e., it must be capable, at least in principle, of test by experiment or observation.

R4: The sentences constituting the explanans must be true.

*Studies in the logic of Explanation(Hempel, Oppenheim, 1948)*

# Problems for Hempel's model

## 1. Accidental Generalizations

**R2:** The explanans must contain general laws, and these must actually be required for the derivation of the explanandum.

**Example (一般规律未应用在推演中)**

(1) 所有自由落体都有恒定加速度

(2) 周一下雨了

所以 (3) 周一下雨了

**Example (无一般规律)**

(1) 这胎生下的所有小狗前额上都有一个褐斑 (2) 费多是这胎生下的一只小狗所以 (3) 费多在其前额有一个褐斑。

**What are general laws?**

逻辑经验论者早就识别出定律的的几个特征，后来有了广泛地共识：

- 定律是如下形式的普遍命题。“所有 A 都是 B”或者“如果事情 E 发生，那么事件 F 总是发生”。
- 定律并不隐含的或明确地指向特定的对象、地点和时间。

**Example (Salmon,1989)**

所有实心球状纯钚块的质量不超过 100 吨。

所有实心球状纯金块的质量不超过 100 吨。

## 2. Irrelevant Premises

**Example**

(1) 所有服用避孕药的男人都不会怀孕。

(2) 小明 (男) 服用了避孕药.

所以 (3) 小明 (男) 不会怀孕.

## 3. Asymmetry

**Example**

Why does this flagpole have a shadow of 10 metres long?

The flagpole is 10 metres high. The sun is at 45° above the horizon.

Because light moves in a straight line, we can derive (by means of the Pythagorean Theorem) that the flagpole has a shadow of 10 metres long.

**Example**

Why is this flagpole 10 metres high?

The flagpole has a shadow of 10 metres long. The sun is at 45° above the horizon.

Because light moves in a straight line, we can derive (by means of the Pythagorean Theorem) that the flagpole is 10 metres high.

# Strategies for Solving Problems

**1. Causal Derivation(Daniel Hausman)**

Hausman's solution is straightforward: only derivations from causes (causal derivations) are explanatory, derivations from effects are not explanatory.

The criterion for distinguishing them is independent alterability:

**Definition (Independent Alterability)**

For every pair of variables, X and Y, whose values are specified in a derivation, if the value of X were changed by intervention, then the value of Y would be unchanged *(Hausman, 1998, p.167).*

# 2. Positive Causal Factors (Nancy Cartwright)

Cartwright requires that the explanans contains causes and that it increases the probability of the explanandum:

**Example**

why did the mayor contract paresis(麻痹性痴呆)?

he had untreated latent syphilis. (隐性梅毒)

7% of the people with latent untreated syphilis get paresis.

## 3. Positive and Negative Causal Factors (Paul Humphreys)

According to Humphreys, singular explanations have the following canonical form:

$Y$ in $S$ at $t$ (occurred, was present) because of $\varphi$, despite $\psi$ (Humphreys, 1989, p.101).

**Example (Why did Albert die?)**

(1)The bubonic plague bacillus (黑死病杆菌) will, if left to develop unchecked in a human, produce death in between 50 and 90% of cases.
(2)It is treatable with antibiotics(抗生素) such as tetracycline(四环素), which reduces the chance of mortality to between 5 and 10%.
(3)Albert contracted plague bacillus and was treated with antibiotics.

# 4. Unificationism (Philip Kitcher)

According to Kitcher, An ideal explanation does not simply list the premises but shows how the premises yield the conclusion.

*Explanatory Unification and the Causal Structure of the World,1989*

- K: A set of beliefs
- A systematization of K: Any set of arguments whose premises and conclusions belong to K.
- Unification: Unification is reached by systematizing our set of beliefs.
- An acceptable argument is an explanation if and only if it instantiates an argument pattern that belongs to a privileged set of argument patterns.

An argument pattern is a triple of

- A sequence of schematic sentences;
- A set of sets of filling instructions;
- A classification.

**Sequence of sentences:**

1. Harry Smith (a) is a member of the Greenbury School Board (P).
2. All members of the Greenbury School Board (P) are bald.
3. Harry Smith (a) is bald.

**Classification:**

(1)and (2) are premises, (3) follows from (1) and (2) by means of universal instantiation and modus ponens.

- Sequence of schematic sentences:

  (1) a is a P.

  (2) All P's are bald.

  (3) a is bald.
- Filling instructions

  (F1): In (1) a must be replaced with the name of an individual, P with an arbitrary predicate.

  (F2): In (2) p must be replaced with the same predicate as in (1).

  (F3): In (3) a must be replaced with the same name of an individual as in (1).
- Classification

  (1) and (2) are premises, (3) follows from (1) and (2) by means of universal instantiation and modus ponens.

# 5. The Causal-Mechanical Model (Wesley Salmon)

*Etiological explanation $\cdots$ involves the placing of the explanandum in a causal network consisting of relevant causal interactions that occurred previously and suitable causal processes that connect them to the fact-to-be-explained. [Salmon, 1984, p.269]*

# Summary of Strategies

| | Hempel | Hausman | Cartwright | Humphreys | Kitcher | Salmon |
|---|---|---|---|---|---|---|
| Are explanation arguments | Yes | Yes | No | No | Yes | No |
| Can explanations contain accidental generalsations | Yes | No | N/A | N/A | No | N/A |
| Can explanations contain irrelevant premises? | Yes | No | N/A | N/a | No | N/A |
| Do explanations cite causes | No | Yes | Yes | Yes | No | Yes |
| Do explanations increase the probability of the explanandum? | Yes | Yes | Yes | No | Yes | No |

## The Logic of Provability (Gödel, 1933)

Gödel suggested a provability reading of modal logic S4, which is axiomatized over the classical logic by the following list of postulates:

- $\Box(F \to G) \to (\Box F \to \Box G)$ Deductive Closure/Normality
- $\Box F \to \Box\Box F$ Positive Introspection/Transitivity
- $\Box F \to F$ Reflection
- $\vdash F \Rightarrow \vdash \Box F$ Necessitation Rule

# The Logic of Proofs (Artemov, 1995,2001)

Axioms and rules of the Logic of Proofs LP are those of classical propositional logic plus axioms

- $s : (F \to G) \to (t : F \to [s \cdot t] : G)$ — Application
- $t : F \to !t : (t : F)$ — Proof Checker
- $s :\to [s + t] : F, t : F \to [s + t] : F$ — Sum
- $t : F \to F$ — Explicit Reflection

# The Logic of Justification (Artemov, 2008)

The celebrated account of Knowledge as Justified True Belief commonly attributed to Plato was widely accepted until 1963 when a paper by Gettier (1963) opened the door to a broad philosophical discussion of the subject.

Justification Logic is based on classical propositional logic augmented by justification assertions $t : F$ that read $t$ is a justification for $F$.

- Justification $\Rightarrow$ Explanation
- Kripke-Fitting model.

# §4 The Logic of Knowledge Why (KW)

**Language:**(single-agent)

$$\varphi = \top \mid p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \mathcal{K}\varphi \mid \mathcal{K}_{why}\varphi$$

- A1 Classical Propositional Axioms
- A2 $\mathcal{K}(\varphi \to \psi) \to (\mathcal{K}\varphi \to \mathcal{K}\psi)$
- A3 $\mathcal{K}_{why}(\varphi \to \psi) \to (\mathcal{K}_{why}\varphi \to \mathcal{K}_{why}\psi)$
- A4 $\mathcal{K}_{why}\varphi \to \mathcal{K}\varphi$
- A5 $\mathcal{K}_{why}\varphi \to \mathcal{K}\mathcal{K}_{why}\varphi$
- A6 $\mathcal{K}\varphi \to \mathcal{K}\mathcal{K}\varphi$
- R1 Modus Ponens
- R2 $\vdash \varphi \quad \Rightarrow \quad \vdash \mathcal{K}\varphi$

# Basic Epistemic Semantics

**Definition (KW-model: $\mathcal{M} = (W, E, R, \mathcal{E}, V)$ )**

W: The set of possible worlds

E: The set of explanations

R: The accessible relation between the worlds in W, R is transitive.

$\mathcal{E}$: $\mathcal{E}(t, \varphi) \subseteq W$ specifies the set of possible worlds where t is considered admissible explanation for $\varphi$.
An admissible explanation function $\mathcal{E}$ must satisfy the conditions: $\forall r, s$, If $w \in \mathcal{E}(r, \varphi \to \psi) \cap \mathcal{E}(s, \varphi)$, then there exists $t$ such that $w \in \mathcal{E}(t, \psi)$ and $v \in \mathcal{E}(t, \psi)$ for all $v$ such that $wRv$ and $v \in \mathcal{E}(r, \varphi \to \psi) \cap \mathcal{E}(s, \varphi)$

V: Atom $\to \mathcal{P}(W)$

Now, we can define the satisfiable relation $\Vdash$:

- $w \Vdash \top$
- $w \Vdash p$ iff $w \in V(p)$
- $w \Vdash \neg\varphi$ iff $w \nVdash \varphi$
- $w \Vdash \varphi \wedge \psi$ iff $w \Vdash \varphi$ and $w \Vdash \psi$
- $w \Vdash \mathcal{K}\varphi$ iff for each $v$ such that $wRv$, $v \Vdash \varphi$
- $w \Vdash \mathcal{K}_{why}\varphi$ iff (1)$\exists t, w \in \mathcal{E}(t, \varphi), \forall v,\ wRv, v \in \mathcal{E}(t, \varphi)$ and (2) $\forall v,\ wRv, v \Vdash \varphi$

# Soundness and Completeness

**Theorem**

*KW is sound and complete for the class of all KW-models.*

**Soundness:**

Induction on derivations in KW. Let us check the axioms.

A3: Suppose $w \Vdash \mathcal{K}_{why}(\varphi \to \psi)$ and $w \Vdash \mathcal{K}_{why}\psi$. Then by the definition of $\Vdash$, we have that $\exists r, w \in \mathcal{E}(r, \varphi \to \psi), v \in \mathcal{E}(r, \varphi \to \psi), v \Vdash \varphi \to \psi, v \Vdash \varphi$ and $\exists s, w \in \mathcal{E}(s, \varphi), v \in \mathcal{E}(s, \varphi)$ for each $v$ such that $wRv$. By the closure conditions of the admissible explanation function, we have that there exists $t, w \in \mathcal{E}(t, \psi)$ and $v \Vdash \psi, v \in \mathcal{E}(t, \psi)$ for each $v$ such that $wRv$. Hence $w \Vdash \mathcal{K}_{why}\psi$.

**Completeness:**

To establish completeness, we use standard canonical model construction. The canonical model $\mathcal{M}^c = (W^c, E^c, R^c, \mathcal{E}^c, V^c)$ for KW is defined as follows:

Let *Form* be the set of all formulas. Define $\Sigma = \{| \ f : Form \rightsquigarrow E^c, f \text{ is a partial function}\}$. For each $f \in \Sigma$, $f$ satisfies the condition as follows: If $f(\varphi \to \psi) = r$ and $f(\varphi) = s$, then there exists $t$ such that $f(\psi) = t$.

- $W^c = \{\langle\Gamma, f\rangle \mid \langle\Gamma, f\rangle \in$MCS$\times\Sigma$, If $\mathcal{K}_{why}\varphi \in \Gamma$, then there exists $t$ such that $f(\varphi) = t\}$, MCS is the set of all maximal consistent sets in KW. Following an established tradition, we denote elements of $W^c$ as $\langle\Gamma, f_\Gamma\rangle, \langle\Delta, g\rangle$, and so forth;
- $\langle\Gamma, f\rangle R^c \langle\Delta, g\rangle$ iff for all formulas such as $\mathcal{K}_{why}\varphi \in \Gamma$, $f(\varphi) = g(\varphi)$ and $\Gamma^{\#} \subseteq \Delta$, where $\Gamma^{\#} = \{\varphi \mid \mathcal{K}_{why}\varphi \in \Gamma\} \cup \{\mathcal{K}_{why}\varphi \mid \mathcal{K}_{why}\varphi \in \Gamma\} \cup \{\varphi \mid \mathcal{K}\varphi \in \Gamma\}$
- $\mathcal{E}^c(t, \varphi) = \{\langle\Gamma, f\rangle \mid \mathcal{K}_{why}\varphi \in \Gamma, f(\varphi) = t\}$
- $V^c(p) = \{\langle\Gamma, f\rangle \mid p \in \Gamma\}$

To prove $R^c$ is transitive:

Suppose $\langle \Gamma, f\rangle R^c\langle \Delta, g\rangle$ and $\langle \Delta, g\rangle R^c\langle \Theta, h\rangle$. $\varphi, \psi$ are arbitrary formulas. Suppose $\mathcal{K}_{why}\varphi, \mathcal{K}\psi \in \Gamma$. By the definition of $R^c$, we have $f(\varphi) = g(\varphi)$ and $g(\varphi) = h(\varphi)$, thus $f(\varphi) = h(\varphi)$. By the axiom $\Vdash \mathcal{K}\psi \to \mathcal{K}\mathcal{K}\psi$ and the properties of MCS, we have $\mathcal{K}\mathcal{K}\psi \in \Gamma$. By the definition of $R^c$, we have $\varphi, \mathcal{K}_{why}\varphi, \mathcal{K}\psi \in \Delta$. Since $\mathcal{K}_{why}\varphi \in \Delta$ and $\langle \Delta, g\rangle R^c\langle \Theta, h\rangle$, we have $\varphi, \mathcal{K}_{why}\varphi \in \Theta$. As $\mathcal{K}\psi \in \Delta$ and $\langle \Delta, g\rangle R^c\langle \Theta, h\rangle$, we have $\psi \in \Theta$. Therefore we conclude that $\langle \Gamma, f\rangle R^c\langle \Theta, h\rangle$ by the definition of $R^c$.

To prove $\mathcal{E}^c$ is well-defined:

Suppose $\langle \Gamma, f \rangle \in \mathcal{E}^c(r, \varphi \to \psi)$ and $\langle \Gamma, f \rangle \in E^c(s, \varphi)$. Then we have $f(\varphi \to \psi) = r$ and $f(\varphi) = s$. By the condition of $f$, we have that there exists $t$ such that $f(\psi) = t$. Thus $\langle \Gamma, f \rangle \in \mathcal{E}^c(t, \psi)$ by the definition of $\mathcal{E}^c$. $\forall \langle \Delta, g \rangle, \langle \Gamma, f \rangle R^c \langle \Delta, g \rangle$. By the denfinition of $R^c$, If $\forall \mathcal{K}_{why}\varphi \in \Gamma$, then $\mathcal{K}_{why}\varphi \in \Delta$ and $f(\varphi) = g(\varphi)$. Thus we have $f(\psi) = g(\psi) = t$. Hence $\langle \Delta, g \rangle \in \mathcal{E}^c(t, \psi)$.

The Truth Lemma claims that for all $\varphi$'s,

$$\langle \Gamma, f \rangle \Vdash \varphi \textit{ if and only if } \varphi \in \Gamma$$

This is established by standard induction on the complexity of $\varphi$. The atomic cases are covered by the definition of $' \Vdash '$. The Boolean induction steps are standard. Consider the case when $\varphi$ is $\mathcal{K}_{why}\psi$ for some $\psi$.

- $\Leftarrow$ If $\mathcal{K}_{why}\psi \in \Gamma$, then $\psi, \mathcal{K}_{why}\psi \in \Delta$ such that $\langle \Gamma, f \rangle R^c \langle \Delta, g \rangle$ by the definition of $R^c$. By the Induction Hypothesis, $\langle \Delta, g \rangle \Vdash \psi$. In addition, $\exists t, \langle \Gamma, f \rangle, \langle \Delta, g \rangle \in \mathcal{E}^c(t, \psi)$ by the definition of $W^c$ and $\mathcal{E}^c$. Hence $\langle \Gamma, f \rangle \Vdash \mathcal{K}_{why}\psi$.
- $\Rightarrow$ If $\mathcal{K}_{why}\psi \notin \Gamma$, then for all $t, \langle \Gamma, f \rangle \notin \mathcal{E}^c(t, \psi)$, Hence $\langle \Gamma, f \rangle \nVdash \mathcal{K}_{why}\psi$.

## Lemma (Existence Lemma)

*For the logic KW and any state* $\langle\Gamma, f\rangle \in W^c$*. If* $\widehat{\mathcal{K}}\varphi \in \langle\Gamma, f\rangle$ *then there is a state* $\langle\Delta, g\rangle \in W^c$ *such that* $\langle\Gamma, f\rangle R^c \langle\Delta, g\rangle$ *and* $\varphi \in \Delta$*.*

**Proof.** Let $\Delta^-$ be

$\{\varphi\} \cup \{\varphi \mid \mathcal{K}\varphi \in w\} \cup \{\mathcal{K}_{why}\psi \mid \mathcal{K}_{why}\psi \in w\} \cup \{\chi \mid \mathcal{K}_{why}\chi \in w\}$. Then $\Delta^-$ is consistent. Suppose not. Then there are $\varphi_1, \cdots, \varphi_m, \mathcal{K}_{why}\psi_1, \cdots, \mathcal{K}_{why}\psi_n, \chi_1, \cdots, \chi_l$ such that

$\vdash_{KW} \varphi_1 \wedge \cdots \wedge \varphi_m \wedge \mathcal{K}_{why}\psi_1 \wedge \cdots \wedge \mathcal{K}_{why}\psi_n \wedge \chi_1 \wedge \cdots \wedge \chi_l \to \neg\varphi.$

$\vdash_{KW} \mathcal{K}(\varphi_1 \wedge \cdots \wedge \varphi_m \wedge \mathcal{K}_{why}\psi_1 \wedge \cdots \wedge \mathcal{K}_{why}\psi_n \wedge \chi_1 \wedge \cdots \wedge \chi_l) \to \mathcal{K}\neg\varphi.$

$\vdash_{KW} (\mathcal{K}\varphi_1 \wedge \cdots \wedge \mathcal{K}\varphi_m \wedge \underline{\mathcal{K}\mathcal{K}_{why}\psi_1 \wedge \cdots \wedge \mathcal{K}\mathcal{K}_{why}\psi_n} \wedge \mathcal{K}\chi_1 \wedge \cdots \wedge \mathcal{K}\chi_l) \to \mathcal{K}(\varphi_1 \wedge \cdots \wedge \varphi_m \wedge \mathcal{K}_{why}\psi_1 \wedge \cdots \wedge \mathcal{K}_{why}\psi_n \wedge \chi_1 \wedge \cdots \wedge \chi_l)$

$$dom(g) = dom(f) \cup \{\varphi \mid \mathcal{K}_{why}\varphi \in \Delta\}, g(\varphi) = \begin{cases} t & \mathcal{K}_{why}\varphi \in \Delta \backslash \Gamma \\ f(\varphi) & \text{otherwise} \end{cases}$$

## KW4: KW+ $\mathcal{K}\varphi \to \varphi$ (A7)

**Definition (KW4-model)**

KW-model with reflexive accessibility relations R.

**Theorem**

*KW4 is sound and complete for the class of all KW4-models.*

**Proof.**

**Soundness:** It is sufficient to prove A7 holds in KW4-models. Trival

**Completeness:** It suffices to check that $R^c$ in the canonical model is reflexive. To prove $\langle \Gamma, f \rangle R^c \langle \Gamma, f \rangle$ for all $\langle \Gamma, f \rangle \in W^c$.

$\forall \mathcal{K}_{why}\varphi, \mathcal{K}\psi \in \Gamma$, by $\vdash \mathcal{K}_{why}\varphi \to \mathcal{K}\varphi$ and the properties of MCS, we have $\mathcal{K}\varphi \in \Gamma$. Similarly, we have $\varphi, \psi \in \Gamma$, as $\vdash \mathcal{K}\varphi \to \varphi$ for any formula $\varphi$. Hence we have $\langle \Gamma, f \rangle R^c \langle \Gamma, f \rangle$ by the definition of $R^c$. □

**KW45: KW+**$\neg\mathcal{K}\varphi \to \mathcal{K}\neg\mathcal{K}\varphi$ **(A8)+**$\neg\mathcal{K}_{why}\varphi \to \mathcal{K}\neg\mathcal{K}_{why}\varphi$ **(A9)**

Definition (KW45-model)

KW-model with reflexive and Euclidean accessibility relations R.

Theorem

*KW45 is sound and complete for the class of all KW45-models.*

**Soundness:** [A8]. Trival. [A9]. Suppose $w \Vdash \neg\mathcal{K}_{why}\varphi$. Then by the definition of $\Vdash$, we have three cases as follow:

- $\forall t, w \notin \mathcal{E}(t,\varphi)$: For each $v$ such that $wRv$, we have $vR^c w$ by $wR^c w$ and the Euclidean property of $R^c$. Thus $v \nVdash \mathcal{K}_{why}\varphi$ (e.g. $v \Vdash \neg\mathcal{K}_{why}\varphi$) for all $v$ such that $wR^c v$. Hence we have $w \Vdash \mathcal{K}\neg\mathcal{K}_{why}\varphi$.
- $\exists v, wR^c v$ and $\forall s, v \notin \mathcal{E}(s,\varphi)$: For each $u$ such that $wR^c u$, we have $uR^c v$ by $wR^c v$ and the Euclidean property of $R^c$. Thus $u \nVdash \mathcal{K}_{why}\varphi$ (e.g. $u \Vdash \neg\mathcal{K}_{why}\varphi$) for all $u$ such that $wR^c u$. Hence we have $w \Vdash \mathcal{K}\neg\mathcal{K}_{why}\varphi$.
- $\exists u, wR^c u$ and $u \nVdash \varphi$. For each $v$ such that $wR^c v$, we have $vR^c u$ by $wR^c u$ and the Euclidean property of $R^c$. Thus $v \nVdash \mathcal{K}_{why}\varphi$ (e.g. $v \Vdash \neg\mathcal{K}_{why}\varphi$) for all $v$ such that $wR^c v$. Hence we have $w \Vdash \mathcal{K}\neg\mathcal{K}_{why}\varphi$.

**Completeness:** It suffices to check that $R^c$ in the canonical model is Euclidean.

Suppose $\langle\Gamma, f\rangle R^c\langle\Delta, g\rangle$ and $\langle\Gamma, f\rangle R^c\langle\Theta, h\rangle$. Then for all formulas such as $\mathcal{K}_{why}\varphi \in \Gamma$, $f(\varphi) = g(\varphi) = h(\varphi)$. For arbitrary $\mathcal{K}_{why}\varphi, \mathcal{K}\psi \in \Delta$, we have $\mathcal{K}_{why}\varphi \in \Gamma$ and $\mathcal{K}\psi \in \Gamma$. Suppose not. By the properties of MCS, we have $\neg\mathcal{K}_{why}\varphi \in \Gamma$ or $\neg\mathcal{K}\psi \in \Gamma$.

- $\neg\mathcal{K}_{why}\varphi \in \Gamma$: By $\neg\mathcal{K}_{why}\varphi \to \mathcal{K}\neg\mathcal{K}_{why}\varphi$ and properties of MCS, we have $\mathcal{K}\neg\mathcal{K}_{why}\varphi \in \Gamma$. Then we have $\langle\Gamma, f\rangle \Vdash \mathcal{K}\neg\mathcal{K}_{why}\varphi \in \Gamma$ by the truth lemma. Then we have $\langle\Delta, g\rangle \Vdash \neg\mathcal{K}_{why}\varphi$. Then we have $\neg\mathcal{K}_{why}\varphi \in \Delta$. Contradiction.
- $\neg\mathcal{K}\psi \in \Gamma$: Similarly, we also get contradiction.

Therefore, it follows that $\mathcal{K}_{why}\varphi, \mathcal{K}\psi \in \Gamma$. Since $\langle\Gamma, f\rangle R^c\langle\Theta, h\rangle$, we have $\mathcal{K}_{why}\varphi, \varphi, \psi \in \Theta$. By the definition of $R^c$, we conclude that $\langle\Delta, g\rangle R^c\langle\Theta, h\rangle$.

Thank you very much for your attention!